Course plan

Subject	**Mathematical Methods in Physics IV**		
Matter	Mathematics		
Degree	Physics		
Study program	469	**Reference no.**	45754
Term	Second term	**Type**	Compulsory
Level	Bachelor degree	**Course/Year**	Second year 2019-2020
ECTS units	6 ECTS		
Language	English		
Lecturer in charge	Javier Negro Vadillo Email: jnegro@fta.uva.es		
Lecturer in charge	Javier Negro Vadillo		
Contact details (E-mail, telephone …)	Email: jnegro@fta.uva.es Phone: 983 42 3040 Office: B212		
Office hours	Please check the timetable		
Department	Física Teórica, Atómica y Óptica		

1. Placement of the subject in the study program

1.1 Context

The subject Mathematical Methods in Physics IV provides the Physics students with basic mathematical techniques on Fourier series and transforms, as well as on Partial Differential Equation, with applications to Physics.

1.2 Relationship with other subjects

This course is related to all those subjects in mathematical methods that the students have taken in previous terms. It contains applications to a number of subjects in the Physics curriculum, such as Theoretical Mechanics, Optics, Electromagnetism, Electronics, Quantum Mechanics, Statistical Mechanics, Fluids Mechanics, Cosmology and Gravitation, and Field Theory.

1.3 Requirements

It is recommendable that the students have acquired the knowledge and capabilities provided by the courses of Linear Algebra and Geometry, Mathematical Analysis and Mathematical Methods in Physics II.

2. Competencies and capabilities

2.1 General

T1. Analysis and synthesis skills.

T2. Organization capability.

T3. Oral and writing communication skills.

T4. Problem solving strategies.

T5. Team work capability.

T6. Autonomous work and learning capabilities.

T7. Skills of adaptation to new mathematical methods.

T8. Capability to apply generic methods to particular scenarios.

T9. Creativity.

2.2 Specific

E1.Capability to deliver a presentation on academic topics and research work.

E2. Capability to get into new fields of study and research.

E3. Capability to work out the necessary approximations to make complicated problems manageable.

E4. Computation skills leading to the development of original software, as well as to the application of conventional software packages.

E5. Capability to apply specialized bibliography on Physics and Mathematics to the development of research work and to problem solving.

E6. Teaching skills at academic level.

E7. Capability to integrate the knowledge from different areas in order to apply it to solve complex problems.

E8. Understanding of the most common mathematical methods, both analytical and numerical ones.

3. Aims

1. Learning to apply Fourier series and transforms to solving Physics problems.
2. Learning the properties and solving methods of partial differential equations of first order.
3. Learning the properties and solving methods of linear partial differential equations of second order.
4. Learning the fundamentals of solitons theory and calculus of variations.

4. Contents

- Chapter 0. *Introduction to partial differential equations of several variables*.
  - Review of fundamental concepts in Calculus and Ordinary differential equations.
  - Motivation: examples relevant to Physics.

- Chapter 1. *Partial differential equations of first order*.
  - Cauchy problem: characteristic curves method.
  - Lagrange method.
  - Weak solutions: the continuity equation and shock waves.

- Chapter 2. *Harmonic analysis: Fourier series and Fourier transform.*
  - Computation of Fourier series.
  - Properties and applications of Fourier series.
  - Computation of Fourier transforms.
  - Properties and applications of the Fourier transform.

- Chapter 3. *Linear second order partial differential equations in two variables.*
  - Classification and canonical forms.
  - Boundary conditions and initial conditions.
  - Some examples relevant to Physics.

- Chapter 4. *Linear second order partial differential equations of the elliptic kind: Laplace equation.*
  - Laplace equation in Cartesian and polar coordinates.
  - Laplace equation with Dirichlet and Neumann boundary conditions.
  - Separation of variables method.
  - Laplace equation in 3 dimensions: the Laplace operator in spherical coordinates.
  - Green's function for the Laplace operator.
  - Poisson's equation.

- Chapter 5. *Linear second order partial differential equations of the parabolic kind: diffusion equation*.
  - The heat equation in one spatial dimension.
  - The heat equation in two spatial dimensions.
  - Method of *variation of constants.*
  - *Eigenfunctions* equation.
  - The heat kernel.

- Chapter 6. *Linear second order partial differential equations of the hyperbolic kind: wave equation*.
  - Wave equation in one spatial dimension. Finite domain.
  - Wave equation in one spatial dimension. Infinite domain.
  - Solution à la d'Alembert. Causality in the wave equation.
  - Solution using the Fourier transform.
  - Green's function for the wave equation.
  - Wave equation in two spatial dimensions. Membrane vibration modes.

- Chapter 7. *Introduction to non-linear partial differential equations and calculus of variations.*
  - Solitons: the kink solution.
  - Calculus of functional variations: Euler-Lagrange equations.

5. Schedule

CHAPTERS	LOAD (ECTS)	TIME PERIOD (approx.)
Ch. 0 and 1	1	10 Feb. – 21 Feb.
Ch. 2	1,5	24 Feb. – 13 March
Ch. 3, 4, 5 and 6	3	16 March – 8 May
Ch. 7	0,5	11 May – 22 May

6. Methodology

- Theoretical and practical in-class lectures.
- Eventually, there will be some practical sessions in the computing class.
- Practical exercises will be given. Problem sheets will be handed in at the end of each chapter.

7. References

-
- G. López, *PDE of First Order and Their Applications to Physics,* World Scientific, 1999.
- Y. Pinchover and J. Rubinstein, *An Introduction to Partial Differential Equations,* Cambridge University Press, 2005.
- A. Pinkus and S. Zafrany, *Fourier Series and Integral Transforms,* Cambridge University Press, 1997.
- G. Arfken, *Mathematical Methods for Physicists,* Academic Press, 2001.
- S.J. Farlow, *Partial Differential Equations for Scientists and Engineers,* J. Wiley & Sons, 1982.
- M.Gelfand and S.V. Fomin, *Calculus of Variations,* Dover, 1963.

8. Time distribution of students' activities

IN-CLASS ACTIVITIES	TIME (h)	OUT-OF-CLASS ACTIVITIES	TIME (h)
Theoretical lectures (T/M)	35	Autonomous individual work.	60
Practical lectures (A)	20	Autonomous team work.	10
Laboratory/computing sessions (L)	5	Preparation of exercises to be handed in.	20
In-class total time	**60**	**Out-of-class total time**	**90**

9. Assessment

PROCEDURE	OVERALL WEIGHT	REMARKS
Two assignments will be offered to be addressed at home.	20%	Compulsory
Final exam. It will contain both practical and theoretical questions to be assessed in a 10 point grading scale.	80%	Compulsory.

ADDITIONAL REMARKS
A minimum grade of 4 out of 10 points in the final exam, as well as a minimum grade of 5/10 in the overall assessment are required in order to obtain a *pass*.

10. Final remarks