

# **THE EVOLUTION OF THE USE OF MATHEMATICS IN CANCER RESEARCH**

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*To our parents, who through their efforts, lessons and understanding forged in us a sense of accomplishment and greatness.*

PJGD, IHR, JR.

*To my wife Araceli, my support, so wonderfully rational, so wonderfully emotive.*

PJGD

*To all our trainees and students, from whom we have received more than they ever expected to give us.*

PJGD, IHR, JR.

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## Preface

This book provides an exhaustive and clear explanation of how statistics and mathematics have been used in cancer research, and seeks to help advanced students of biostatistics and biomathematics as well as cancer researchers to achieve their objectives. To do so, state-of-the-art biostatistics and biomathematics are described and discussed in detail through illustrative and capital examples taken from cancer research work already published. The crossed examination of the statistical, mathematical and computational issues arising from the selected examples redounds to a didactic, homogeneous and unified vision of the application of statistics and mathematics in biomedicine, especially to the study of cancer, and illustrates the capability of these logical sciences in biomedical research. As a result, the book provides a guide for cancer researchers in using statistics and mathematics, clarifying the contribution of these logical sciences to the study of cancer, thoroughly explaining their procedures and methods, and providing criteria to their appropriate use.

Indeed, this book is designed for advanced students and researchers pursuing the use of biostatistics and biomathematics in their investigations and research in biology and medicine in general, and in cancer in particular. The main virtue of the book is the follow-through that is available by reading the different examples, in a relevant and timely reading that facilitates the understanding of the key aspects underlying the applications of statistics and mathematics in biomedicine, and that provides complete coverage of the most relevant issues in biostatistics and biomathematics. Each chapter has been conceived as a part in the whole in such a way that information flows easily, on the one hand explaining in a concise and clear way a particular subject, and on the other connecting its results with those in the previous and following chapters. Thanks to the use of selected and relevant examples taken from the scientific literature on cancer research, the result is a self-contained book on medicine, statistics and mathematics, which illustrates the potential of biostatistics and biomedicine in biomedical research. Focusing on the achievements that biostatistics and biomathematics have already obtained, researchers can perceive the high returns that the use of statistics and mathematics yield in cancer research, and thanks to the detailed discussion of the applied statistical and mathematical techniques, they can deduce the criteria and motif for finding the appropriate use of these formal disciplines.

The primary audience of the book is advanced undergraduate students and graduate students in medicine and biology, and cancer researchers who seek to learn how statistics and mathematics can help in their future research. We assume no advanced knowledge of statistics and mathematics beyond the undergraduate level. However, the reader should have a minimum formation in these disciplines and be familiar with the contents of undergraduate textbooks on mathematical analysis and biostatistics.

The use of statistics and mathematics in biology and medicine is today increasing, and already forms part of the core of both theoretical and empirical biomedical research. We hope with this book to contribute to a better comprehension of the procedures, methods, criteria and applications of biostatistics and biomathematics in medicine, especially in cancer research.

The authors



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